From $\alpha \approx 55-60$ deg, the forebody vortices are asymmetric and stable with the left vortex assuming a high position regardless of the history of sideslip. At $\alpha$'s among 60 and 65 deg, the flow becomes nearly "bistable." The steady vortical flow tends to assume one of two mirror-image asymmetric flow patterns at a given combination of $\alpha$ and $\beta$, depending on the sideslip history and imposed transient disturbances.[5] The effect of a nose-boom on the baseline F/A-18 flow is illustrated by the examples in Fig. 3. At $\alpha = 50$ deg, shown in Fig. 3a, the nose-boom causes the forebody vortices to become asymmetric. At $\alpha = 55$ and 60 deg, shown in Figs. 3b and 3c, respectively, the natural asymmetry is essentially eliminated. At $\alpha = 65$ deg, shown in Fig. 3d, the asymmetry is reduced but not eliminated.

Therefore, at moderate-to-high angles of attack where the vortex pattern over the nose-boom becomes asymmetric, the wake of a nose-boom seemingly overrides other small naturally present perturbations on the forebody. This can lead either to an increase or a decrease in the natural zero-sideslip forebody vortex asymmetry, depending on the state of the baseline flow. At very high angles of attack where unsteady vortex shedding occurs off the nose boom, the nose-boom wake flow is symmetric on the time-average basis. Evidently, because the shedding frequency of the nose-boom vortices is high compared to the response of the forebody vortex system to imposed disturbances, no significant oscillation of the forebody vortex flow can be observed. The forebody vortex flow is, therefore, essentially symmetric and steady. The main effect of the nose-boom at very high angles of attack would seem to be to reduce forebody vortex asymmetry regardless of the state of the baseline flow.

## IV. Conclusions

The effect of a nose-boom on forebody vortex flows has been studied in a water tunnel using several different model configurations. The wake flow of a nose-boom was observed to be similar to that of highly slender cylindrical bodies. The wake consists of symmetric vortices at moderate angles of attack, asymmetric vortices at high angles of attack, and unsteady vortex shedding at very high angles of attack. The influences of these nose-boom-induced vortex-flows are typically strong compared to other natural sources of forebody vortex asymmetry that may be present. The nose-boom, therefore, has a dominating effect on the forebody vortex asymmetry in most situations. At moderate-to-high angles of attack, the net effect can be a reduction or increase in the zero-sideslip forebody vortex asymmetry, depending on the degree of symmetry of the baseline forebody flow without the nose-boom. At very high angles of attack, the typically high vortex-shedding frequency and the comparatively slow response of the forebody vortices to imposed disturbances result in the elimination or reduction of forebody vortex asymmetry.

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# Technical Comments

## Comment on "NASA Investigation of a Claimed 'Overlap' Between Two Gust Response Analysis Methods"

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THE subject paper[1] provides an interesting and timely comparison of the statistical discrete gust (SDG) and power spectral density (PSD) methods of calculating gust response of aerospace vehicles. Discrete gust methods have a long and honorable history. They have served aviation well for more than half a century,[2] both because of their simplicity, and because they do, indeed, model real events in the atmosphere to a degree that has engineering utility.

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The use of a superposition of discrete gusts to model continuous atmospheric turbulence is, however, a quite different matter. The comment attributed in the paper to J. G. Jones ". . . the former is essentially simply an approximate representation of the latter" is a fair assessment. The paper bears out this assertion in that the claimed equivalence is shown to exist approximately for a special case—that is, input of only vertical gusts to a linear system. One must, however, ask the question, "Why use an approximation when the real thing is available?" Clearly, not to save computing time. The paper shows that the computational cost of the SDG method is more than twenty times that of the PSD method. The reason given in the paper for using SDG is that it provides simultaneous values of many responses (loads) from which practical test loads can be found. This judgment about the PSD method (more properly, conventional random process analysis) is not strictly true. Within the commonly used assumptions of linear systems and Gaussian turbulence, one can, in fact, very easily determine sets of simultaneous loads that exist at any chosen probability level. This is demonstrated below. Thus, this feature of the SDG method is not its main advantage over PSD. The advantage it does have, one that may even be worth the penalty in computing cost, is that it can use non-Gaussian turbulence inputs, because it is well known that atmospheric turbulence has this characteristic. At the same time, certain deficiencies of SDG should be noted. As described, for example in Ref. 3, it makes no provision for variation of vertical gust across the span, nor for simultaneous multiple correlated inputs. These features raise serious doubts about its utility for finding asymmetric loads associated with lateral response modes.

Now to the proof of the above assertion about conventional random process theory. These theories, for linear systems, readily provide for the calculation of all the response power and cross spectra (see Ref. 4, p. 94). For example, consider a case such as those treated in the subject paper; that is, one input (vertical gust) and several responses. Let $x(t)$ denote the input, $y_i(t)$ the $i$th response, and $G_i(\omega)$ the transfer function from $x$ to $y_i$. Then the cross-spectral density between the "principal" response $y_1$ and some other response $y_2$ is

$$\phi_{12}(\omega) = G_1^*(\omega)\phi_{xx}(\omega)G_2(\omega) \tag{1}$$

Here * denotes complex conjugate, and $\phi_{xx}$ is the PSD of $x$. From this cross-spectrum, one gets the covariance of $y_1$ and $y_2$, namely

$$R_{12} = \langle y_1 y_2 \rangle = \int_{-\infty}^{\infty} \phi_{12}(\omega)\, \mathrm{d}\omega \tag{2}$$

Up to this point, the assumption of a Gaussian input has not yet been invoked. That comes next. The knowledge of $R_{12}$, and of the mean-square values $\sigma_i^2 = \langle y_i^2 \rangle$, then permits, for Gaussian variables, the calculation of the joint probability density of $y_1$ and $y_2$ (Ref. 4, page 32), that is

$$f(y_1, y_2) = \frac{1}{2\pi(\sigma_1^2\sigma_2^2 - R_{12}^2)^{1/2}} \times \exp\left[-\frac{1}{2}\frac{\sigma_2^2 y_1^2 - 2R_{12}y_1y_2 + \sigma_1^2 y_2^2}{\sigma_1^2\sigma_2^2 - R_{12}^2}\right] \tag{3}$$

In the application of statistical methods, at some point one must assign an arbitrary (but logically chosen!) probability of occurrence. In this case, one would use the cumulative probability $P(y_1)$ of the main load and choose a large enough value to generate a safe design load $y_{1_d}$ (it might, for example, be $3\sigma_1$). The probability that $y_1$ will exceed $y_{1_d}$ is then $1 - P(y_{1_d})$. In order to choose the design value of $y_2$, one might proceed as follows.

Let $r$ be the fraction of the occurrences of $y_1 \geq y_{1_d}$, at which we simultaneously have $y_2 \geq y_{2_d}$. Then

$$r = \frac{1}{1 - P(y_{1_d})} \int_{y_{1_d}}^{\infty} \mathrm{d}_{y_1} \int_{y_{2_d}}^{\infty} f(y_1, y_2)\, \mathrm{d}y_1\, \mathrm{d}y_2 \tag{4}$$

and is a function of $y_{1_d}$ and $y_{2_d}$. The integral in Eq. (4) is readily evaluated, and for the previously chosen $y_{1_d}$ leads to a monotonic function of $y_{2_d}$. By choosing $r$ (for example, $r = 0.01$) one finds $y_{2_d}$. Thus, the load pair $(y_{1_d}, y_{2_d})$ is obtained with a specified probability of occurrence. In this example, for every 100 occurrences of a primary load $\geq y_{1_d}$, there would be one simultaneous occurrence of a secondary load $\geq y_{2_d}$. The extension to more than two loads is quite straightforward.

As remarked previously, the above analysis is subject to two very important constraints: linearity of the system model and Gaussian turbulence. There is a way to release both of these constraints that may be practical now, or with further evolution of computers, may become practical in the near future. That method is simulation in the time domain. In that technique, nonlinear systems and non-Gaussian turbulence are readily accommodated. When choosing a method for the realization of atmosphere turbulence in a time series, an adaptation of the SDG method is one candidate among several. Realizations of this kind have been used for decades in flight simulators, for example, and have been highly developed for that application.[5] In Ref. 5, Robinson and Reid have compared time series generated by an adaptation of the SDG method with several others for application to flight simulation. They found that the computing cost for the SDG time series was similar to that for "classical" turbulence realizations. The realizations they studied are capable of providing multiple inputs ($u_g$, $v_g$, $w_g$, $p_g$, $q_g$, $r_g$) simultaneously with correct auto- and cross-correlations, and consequently correct power and cross-spectra. The SDG method does not seem to have this capability. The non-Gaussian kurtosis can also be varied. With such methods, one can now visualize the practical calculation of the response of nonlinear elastic airplanes to flight in a three-dimensional non-Gaussian atmosphere. Rational structural loadings could be deduced from the outputs of such computations, and would seem to be a worthy candidate for consideration by the FAA committee referenced in the paper. It would be expected that only a few standard input atmospheres would be needed for this application, so that the computing cost of generating them would be relatively unimportant. Any airplane could then be flown through the appropriate standard turbulence time-history to collect the relevant loading statistics. Such a procedure would yield the additional benefit of providing information on fatigue and controls.

The simulation method would, in a sense, share the philosophical basis of the mission-analysis method described in Ref. 2, except that it would allow for non-Gaussian/nonlinear cases. The crucial practical question relates to the time required to compute one case. In the mission analysis method, the design load sought is that which is exceeded once in about 50,000 h of flying time. If the spectrum of the subject paper is used, we see that the spectral density is down from its peak value by about 2 orders of magnitude when $\omega \doteq 30$ rad/s. If we convert this, for example, to the case L = 2500 ft and V = 1000 fps, then L/V = 2.5, and the corresponding $\omega$ is 30/2.5 = 12 rad/s, which corresponds to about 2 Hz. This upper limit on frequency would permit the inclusion of many elastic modes. If such a frequency had to be accommodated, one would have to compute system updates approximately four times per second of real time. The number of updates for $10^5$ h is then about $1.5 \times 10^9$. This might be reduced by one or two orders of magnitude by eliminating periods of low or zero turbulence. Whether the computation is feasible for the elastic systems of many degrees of freedom that occur in practice is a matter for study. If only rigid-body modes were of interest, the upper limit on frequency could be lower, and the total real time simulated might be further reduced by applying some analytical model for extreme value statistics. Such considerations might lead to sufficient reduction of computing time as to bring it within the capabilities that currently exist.

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